

# Is the anisotropy of the upper critical field of $\text{Sr}_2\text{RuO}_4$ consistent with a helical $p$ -wave state?

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We calculate the angular and temperature  $T$  dependencies of the upper critical field  $H_{c2}(\theta, \phi, T)$  for the  $C_{4v}$  point group helical  $p$ -wave states, assuming a single uniaxial ellipsoidal Fermi surface, Pauli limiting, and strong spin-orbit coupling that locks the spin-triplet  $\mathbf{d}$ -vectors onto the layers. Good fits to the  $\text{Sr}_2\text{RuO}_4$   $H_{c2,a}(\theta, T)$  data of Kittaka *et al.* [2009 *Phys. Rev. B* **80**, 174514] are obtained. Helical states with  $\mathbf{d}(\mathbf{k}) = \hat{k}_x\hat{\mathbf{x}} - \hat{k}_y\hat{\mathbf{y}}$  and  $\hat{k}_y\hat{\mathbf{x}} + \hat{k}_x\hat{\mathbf{y}}$  (or  $\hat{k}_x\hat{\mathbf{x}} + \hat{k}_y\hat{\mathbf{y}}$  and  $\hat{k}_y\hat{\mathbf{x}} - \hat{k}_x\hat{\mathbf{y}}$ ) produce  $H_{c2}(90^\circ, \phi, T)$  that greatly exceed (or do not exhibit) the four-fold azimuthal anisotropy magnitudes observed in  $\text{Sr}_2\text{RuO}_4$  by Kittaka *et al.* and by Mao *et al.* [2000 *Phys. Rev. Lett.* **84** 991], respectively.

## INTRODUCTION

Despite two decades of extensive studies the detailed structure of the superconducting order parameter in  $\text{Sr}_2\text{RuO}_4$  remains unclear [1–3]. Nuclear magnetic resonance (NMR) and nuclear quadrupole resonance (NQR) Knight shift measurements of the electronic spin susceptibility of the O [4, 5] and Ru [6–8] nuclear sites, internal magnetic field measurements by spin-polarized neutron scattering [9, 10] and spin-relaxation measurements by muon spin resonance ( $\mu\text{SR}$ ) [11] all provided support to a parallel-spin pairing state. The invariance of the spin susceptibility on entering the superconducting state with the magnetic field  $\mathbf{H}$  both parallel and perpendicular to the  $\text{RuO}_2$  layers was argued to be consistent with very weak spin-orbit coupling in  $\text{Sr}_2\text{RuO}_4$ , so that the  $\mathbf{d}$ -vector representing the orientation of the spin-triplet pairing state would always rotate to be perpendicular to  $\mathbf{H}$  for  $\mu_0 H > 20$  mT [7], where  $\mu_0$  is the vacuum magnetic permeability. However, this scenario is in direct conflict with the suppression of the in-plane upper critical field  $H_{c2,ab}$  ( $\sim 1.5$  T) at low temperatures  $T$  [12–14], reminiscent of the strong Pauli pairbreaking limit in spin-singlet pair states [15] or a spin-triplet pair state with the  $\mathbf{d}$ -vector parallel to the field [16]. Indeed, with the assumption that the  $\mathbf{d}$ -vector is locked in some direction in the basal plane, the suppression of  $H_{c2,ab}$  could possibly be explained by the inclusion of Pauli pairbreaking [17, 18]. The discrepancies in the orientation of the  $\mathbf{d}$ -vector are even aggravated by the extreme sensitivity of the  $H_{c2,ab}$  suppression [13] as well as by the in-plane anisotropy of  $H_{c2,ab}(\phi)$  [19, 20] to the precise field alignment. Although introducing a multi-component order parameter seems rather unconvincing that it might apply to all cases [21, 22], it might be relevant to the chiral-nonchiral transition in vortex states [23, 24] or even to the first-order transition to the normal state [14]. Further

complicating matters, one set of scanning tunneling microscopy (STM) experiments was consistent with a single nodeless gap on all three Fermi surfaces of  $\text{Sr}_2\text{RuO}_4$  [25], but in another STM experiment, the tip was placed in a spot with substantial normal regions for  $T \ll T_c$  [26], completely disguising any possible superconducting order parameter form. To gain a possibly consistent interpretation to all pieces of experimental evidence, it appears indispensable to introduce a new mechanism to describe the nontrivial interaction between spin-triplet superconductivity and  $\mathbf{H}$ . Beforehand, one could nevertheless assume that the Pauli limit was essential to determine the in-plane  $H_{c2,ab}$ . In addition, since many examples of anomalous Knight shift results in singlet-spin layered and heavy fermion superconductors have been obtained, a new theory of the Knight shift is sorely needed [27, 28].

The possible spin-triplet  $p$ -wave states for  $\text{Sr}_2\text{RuO}_4$  are limited by the tetragonal crystal structure with two-dimensional square lattice point group symmetry  $C_{4v}$  to the six degenerate states with the  $\mathbf{d}$ -vectors  $\hat{k}_x\hat{\mathbf{x}} \pm \hat{k}_y\hat{\mathbf{y}}$ ,  $\hat{k}_y\hat{\mathbf{x}} \pm \hat{k}_x\hat{\mathbf{y}}$  and  $(\hat{k}_x \pm i\hat{k}_y)\hat{\mathbf{z}}$  [30, 31]. The two chiral states  $\mathbf{d} = (\hat{k}_x \pm i\hat{k}_y)\hat{\mathbf{z}}$  with  $\mathbf{d} \parallel \hat{\mathbf{c}}$  are believed to be stabilized near  $\mathbf{H} = 0$  [1, 3], while with  $H \sim H_{c2,ab}$ , only the four helical states with  $\mathbf{d}$ -vectors lying in the basal plane could be consistent with the in-plane  $H_{c2,ab}$  measurements [13] by including the effects of Pauli limiting [18]. Contrary to the the assumption of very weak spin-orbit coupling, allowing the  $\mathbf{d}$ -vector to rotate to a direction perpendicular to  $\mathbf{H}$ , that was argued to explain the Knight shift observations for both  $\mathbf{H} \parallel \hat{\mathbf{c}}$  and  $\mathbf{H} \perp \hat{\mathbf{c}}$ , sufficiently strong spin-orbit coupling should be assumed to allow for Zeeman energy splitting in spin-triplet pairing states [29]. In this case, the degeneracy in the four helical states is lifted [2], since each state responds differently to  $\mathbf{H}$ , as illustrated in figure 1, two of them manifesting themselves by showing intrinsic four-fold in-plane anisotropies of  $H_{c2,ab}(\phi)$  — a novel scenario other

than earlier postulations of a multi-component order parameter [21] or the possible misalignment of two domains in the sample [13]. In this paper, we will calculate the full angular and  $T$  dependencies of  $H_{c2}(\theta, \phi, T)$  for the four helical states to try to set further restrictions on the possible pairing symmetries in  $\text{Sr}_2\text{RuO}_4$ .

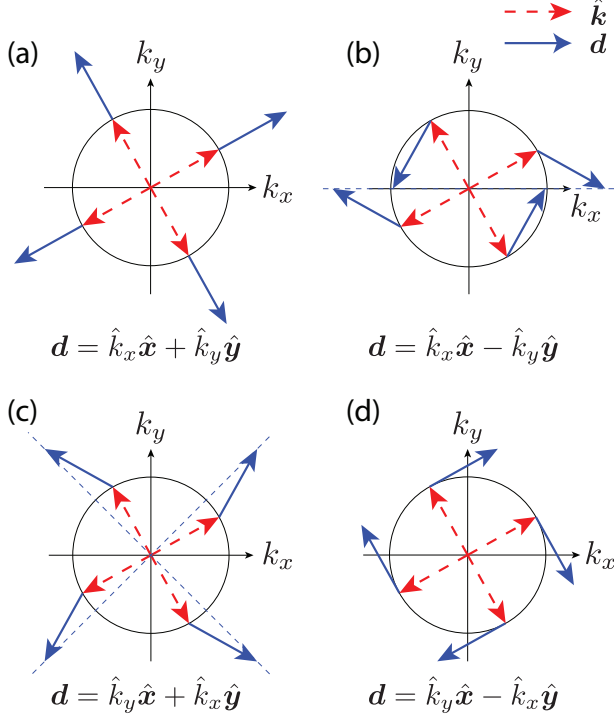


FIG. 1. Illustration of the  $d$ -vectors for the helical states. In terms of  $H_{c2,ab}(\phi)$ , helical states shown in (a) and (d) are isotropic, while those in (b) and (c) exhibit four-fold in-plane anisotropies due to the Pauli paramagnetic effect and strong spin-orbit coupling.

## MODEL

The Fermi surface of  $\text{Sr}_2\text{RuO}_4$  consists of three sheets: a quasi-two-dimensional  $\gamma$  band, and a pair of quasi-one-dimensional ( $\alpha$ ,  $\beta$ ) bands [32]. Although still under debate [33–35], the cylindrical  $\gamma$  band is widely considered to be the primary source of  $p$ -wave pairing [3, 36]. The small  $c$ -axis dispersion in this nearly cylindrical  $\gamma$  Fermi surface can be incorporated by treating it as an elongated uniaxial ellipsoid, characterized by the effective mass anisotropy of the quasi-particles  $m_c \gg m_a = m_b = m_{ab}$ . The primary pair-breaking effects established in superconductivity fall into two categories: 1. the orbital effect arising from the competition between the coherence of two quasi-particles in a Cooper pair and their individual orbital motions in a magnetic field, i.e., the Landau levels governed by the effective vector potential  $\mathbf{A}$  [37]; 2. the paramagnetic effect due to the Zeeman energy

gained from the interactions between their spins and the field [15]. Highly anisotropic Zeeman interactions are expected in the layered compound  $\text{Sr}_2\text{RuO}_4$ , described here by an effective diagonal  $g$ -tensor  $g = \text{diag}(g_a, g_b, g_c)$  with  $g_c \neq g_a = g_b = g_{ab}$  as in  $-\mu_B \mathbf{S} \cdot \mathbf{g} \cdot \overline{m}_{1/2} \cdot \mathbf{B}$ , where  $\mu_B$  is the Bohr magneton,  $\mathbf{S}$  is the electron spin,  $\overline{m}_{1/2} = \text{diag}(\overline{m}_a^{1/2}, \overline{m}_b^{1/2}, \overline{m}_c^{1/2})$  is the diagonal tensor of the square roots of the relative effective masses  $\overline{m}_\mu = m_\mu/m$  ( $\mu = a, b, c$ ) with geometric mean effective mass  $m = (m_a m_b m_c)^{1/3}$ , and the magnetic induction  $\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M} = \nabla \times \mathbf{A}$ , where  $\mathbf{M}$  is the magnetization proportional to  $\mathbf{H}$  for the non-ferromagnetic superconductor  $\text{Sr}_2\text{RuO}_4$  [11]. If the  $d$ -vector is along the  $c$  axis, neither the chiral Anderson-Brinkman-Morel (ABM) state  $\Delta_0(\hat{k}_x + i\hat{k}_y)\hat{z}$  [38, 39] nor the Scharnberg-Klemm (SK) state  $[\Delta_{0+}(\hat{k}_x + \hat{k}_y) + \Delta_{0-}(\hat{k}_x - i\hat{k}_y)]\hat{z}$  [24, 40], nor generalizations of them obtained by setting  $\hat{k}_x \rightarrow \sin(k_x a)$  and  $\hat{k}_y \rightarrow \sin(k_y a)$  [26], could fit the in-plane  $H_{c2,ab}$  measurements; for comparison, even the conventional  $s$ -wave state without Pauli limiting has  $H_{c2}(T)$  well above the experimental data of Kittaka *et al.* for  $\mathbf{H} \parallel \hat{a}$  (figure 2). Instead, the helical states have a chance for Pauli limiting to play the crucial role in suppressing  $H_{c2,ab}(T)$ , as long as  $\mathbf{H}$  cannot cause the  $d$ -vectors to rotate.

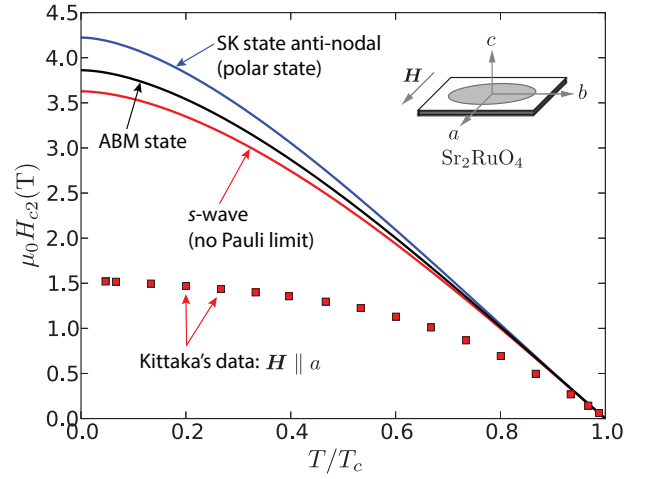


FIG. 2. Fits of the chiral ABM, SK and  $s$ -wave (without Pauli limiting) states to the in-plane  $H_{c2,a}(T)$  measurements of  $\text{Sr}_2\text{RuO}_4$  [13]. The in-plane  $H_{c2,ab}(T)$  is strongly suppressed at low temperatures from that predicted from the orbital pairbreaking in these states. Note that in the anti-nodal direction,  $H_{c2,ab}(T)$  of the chiral SK state has a first-order transition to that of the nonchiral polar state  $\mathbf{d} = \Delta_0 k_x \hat{z}$  [24].

Hence we model  $\text{Sr}_2\text{RuO}_4$  as a clean homogeneous weak-coupling type-II superconductor. Since close to  $H_{c2}$ ,  $\Delta(\mathbf{R}) = \sum_{n=0}^{\infty} a_n |n(\mathbf{R})\rangle$  for the vortex lattice in the mixed state, is constructed from the harmonic oscillator states  $|n\rangle$ , and is vanishingly small, the Gor'kov equations for  $p$ -wave superconductors with a single ellipsoidal Fermi surface can be linearized and transformed

to yield [40, 41, 44]

$$\begin{aligned} \Delta(\mathbf{R}) = & 2\pi T N(0) V_0 \sum_{\omega_n} \int \frac{d\Omega_{\mathbf{k}'}}{4\pi} \int_0^\infty d\xi \exp[-2|\omega_n|\xi] \\ & \times \exp\left[-i \operatorname{sgn}\omega_n \xi \mathbf{v}_F(\hat{\mathbf{k}}') \cdot (\alpha \nabla_{\mathbf{R}}/i + 2e\mathbf{A})\right] \Delta(\mathbf{R}) \\ & \times \left(|\mathbf{d}(\hat{\mathbf{k}}')|^2 + [\cos(\alpha g_{\text{eff}} \mu_B B \xi) - 1] |d_z(\hat{\mathbf{k}}')|^2\right), \quad (1) \end{aligned}$$

where  $N(0)$  is the density of states per spin at the Fermi level,  $V_0$  is the pairing amplitude,  $\omega_n$  are the fermion Matsubara frequencies,  $\mathbf{v}_F = \mathbf{k}_F/m$  is the effective Fermi velocity,  $\alpha = (\overline{m}_{ab} \sin^2 \theta + \overline{m}_c \cos^2 \theta)^{1/2}$  characterizes the geometric anisotropy of the Fermi surface,  $g_{\text{eff}} = [g_c^2 \cos^2 \theta' + g_{ab}^2 \sin^2 \theta']^{1/2}$  is the effective  $g$ -factor experienced by the spins with  $\cos \theta' = \sqrt{\overline{m}_c} \cos \theta / \alpha$ ,  $e$  is the electronic charge and the convention  $\hbar = c = k_B = 1$  is adopted. We note that the Klemm-Clem (KC) transformations have been performed so that the  $\hat{\mathbf{z}}'$  direction in (1) is always along  $\mathbf{B}'$  [42–44].

All of the helical states in figure 1 are degenerate in terms of the KC transformed  $|\mathbf{d}(\hat{\mathbf{k}}')|^2 = (\hat{k}'_x \cos \theta' + \hat{k}'_z \sin \theta')^2 + \hat{k}'_y{}^2$ . However, the  $z$ -components  $|d_z(\hat{\mathbf{k}}')|^2 = |\hat{\mathbf{B}}' \cdot \mathbf{d}(\hat{\mathbf{k}}')|^2$ , which contribute to the Zeeman energy, are distinct for each of the four helical states. For the helical state  $\mathbf{d} = \hat{k}_x \hat{\mathbf{x}} - \hat{k}_y \hat{\mathbf{y}}$  in figure 1(b), the KC transformed

$$|d_z(\hat{\mathbf{k}}')|^2 = (g_{ab}/g_{\text{eff}})^2 \sin^2 \theta' [(\hat{k}'_x \cos \theta' + \hat{k}'_z \sin \theta') \cos 2\phi - \hat{k}'_y \sin 2\phi]^2, \quad (2)$$

is anisotropic in the basal plane [42, 43], where  $\phi' = \phi$  for  $m_a = m_b$ , and for consistency we set  $\hat{k}'_y \rightarrow \hat{k}'_x$  and  $\hat{k}'_x \rightarrow -\hat{k}'_y$ .  $|d_z(\hat{\mathbf{k}}')|^2$  in state (c) is obtained from that of helical state (b) in (2) by letting  $\phi \rightarrow \phi - \pi/4$ , while  $|d_z(\hat{\mathbf{k}}')|^2$  for the helical states (a) and (d) are respectively obtained by setting  $\phi = 0$  and  $\phi = \pi/4$  in (2). These latter two helical states are therefore isotropic in the basal plane. Accordingly, the helical state (b) with  $\mathbf{d} = \hat{k}_x \hat{\mathbf{x}} - \hat{k}_y \hat{\mathbf{y}}$  can be used to present the formulation.

We introduce the dimensionless quantities  $t = T/T_c(0)$ ,  $b_{c2} = B_{c2}/B_0$  and for the  $g$ -tensor (via its elements)  $\bar{g} = g/g_0$ , where  $T_c(0) = (2e^C \omega_c / \pi) e^{-1/N(0)V_0}$  is the superconducting transition temperature in zero field,  $C = 0.577$  is the Euler constant,  $\omega_c$  is the energy cutoff from the BCS theory,  $B_0 = [2\pi T_c(0)]^2 / 2ev_F^2$  and  $g_0 = 2\pi T_c(0) / \mu_B B_0$ . Equation (1) can be expanded as [40]

$$[-\ln t + \alpha_n^{(p)} + \alpha_n^{(a)}] a_n + \beta_{n-2}^{(+)} a_{n-2} + \beta_n^{(-)} a_{n+2} = 0. \quad (3)$$

The upper critical field  $b_{c2}$  is embedded in the coefficients

$$\begin{aligned} \alpha_n^{(p)} = & \int_0^\infty d\psi \frac{\sinh^2 \psi}{\cosh^3 \psi} \int_0^\infty d\rho \frac{t \sqrt{2/\alpha b_{c2}}}{\sinh(t \sqrt{2/\alpha b_{c2}} \rho \cosh \psi)} \\ & \times \left[ e^{-\frac{1}{2}\rho^2} L_n^{(0)}(\rho^2) F^{(p)} - \sin^2 \theta' \right], \quad (4) \end{aligned}$$

$$\begin{aligned} \alpha_n^{(a)} = & \int_0^\infty d\psi \frac{1}{2 \cosh^3 \psi} \int_0^\infty d\rho \frac{t \sqrt{2/\alpha b_{c2}}}{\sinh(t \sqrt{2/\alpha b_{c2}} \rho \cosh \psi)} \\ & \times \left[ e^{-\frac{1}{2}\rho^2} L_n^{(0)}(\rho^2) F^{(a)} - (1 + \cos^2 \theta') \right], \quad (5) \end{aligned}$$

$$\begin{aligned} \beta_n^{(\pm)} = & \int_0^\infty d\psi \frac{1}{4 \cosh^3 \psi} \int_0^\infty d\rho \frac{t \sqrt{2/\alpha b_{c2}}}{\sinh(t \sqrt{2/\alpha b_{c2}} \rho \cosh \psi)} \\ & \times e^{-\frac{1}{2}\rho^2} \frac{-\rho^2}{\sqrt{(n+2)(n+1)}} L_n^{(2)}(\rho^2) \\ & \times \left[ -\sin^2 \theta' + G(\cos^2 \theta' \cos^2 2\phi - \sin^2 2\phi) \right. \\ & \left. \pm i G \cos \theta' \sin 4\phi \right], \quad (6) \end{aligned}$$

where the  $L_n^{(m)}$  are the associated Laguerre polynomials, and

$$F^{(p)} = \sin^2 \theta' + G \sin^2 \theta' \cos^2 2\phi, \quad (7)$$

$$F^{(a)} = 1 + \cos^2 \theta' + G(\cos^2 \theta' \cos^2 2\phi + \sin^2 2\phi) \quad (8)$$

with  $G = [\cos(\bar{g}_{\text{eff}} \sqrt{\alpha b_{c2}/2}) - 1] (\bar{g}_{ab}/\bar{g}_{\text{eff}})^2 \sin^2 \theta'$ . The solution to (3) constitutes the determinant of the (infinite order) tridiagonal matrix constructed from the coefficients of  $a_n$  ( $n = 0, 2, \dots$ ), which can be solved numerically for arbitrary  $t$ . To calculate  $B_{c2} = \mu_0 H_{c2}$  for non-magnetic superconductors, usually the first 3 or 4 orders produce sufficiently accurate results to show all of the essential features.

## RESULTS

Figure 3 shows our fits to the angular dependent  $H_{c2,a}(\theta, T)$  measurements of Kittaka *et al.* on a sample of  $\text{Sr}_2\text{RuO}_4$  ( $T_c(0) = 1.503$  K) [13] using helical state (b) with  $\mathbf{d} = \hat{k}_x \hat{\mathbf{x}} - \hat{k}_y \hat{\mathbf{y}}$ . The appropriateness of an elongated uniaxial ellipsoidal Fermi surface for the  $\gamma$  band is verified by the huge effective mass anisotropy  $m_c/m_{ab} = 1067$  estimated from the slopes of  $H_{c2,a}(T)$  at  $T_c(0)$  in the [100] and [001] crystal directions where Pauli limiting effects are negligible. Down to low  $t$ , a suitable choice of the effective  $g$ -factor will further suppress the  $H_{c2}$  curves, especially for those with  $\theta < 5^\circ$  (c.f. figure 2). Although the  $H_{c2,a}(T, \theta > 5^\circ)$  data appear to follow the anisotropic effective mass model [13, 24, 44], one should nevertheless take into consideration the intrinsic anisotropy of  $H_{c2}(\theta)$  raising from the point nodal structures of the helical states ( $[H_{c2,ab}/H_{c2,c}]_{T \rightarrow T_c(0)} = \sqrt{2}$  for an isotropic Fermi surface) [24]. For an overall best fit, the effective  $g$ -tensor was evaluated to have the diagonal elements

$\bar{g}_c = 0.2$  and  $\bar{g}_{ab} = 1.9$ . Obviously, the small-valued  $\bar{g}_c$  doesn't contribute to  $H_{c2,c}$  since  $\mathbf{d} \perp \hat{\mathbf{c}}$  for the helical states, but it plays a role in determining  $H_{c2}(\theta)$  for  $(0^\circ < \theta < 90^\circ)$ .

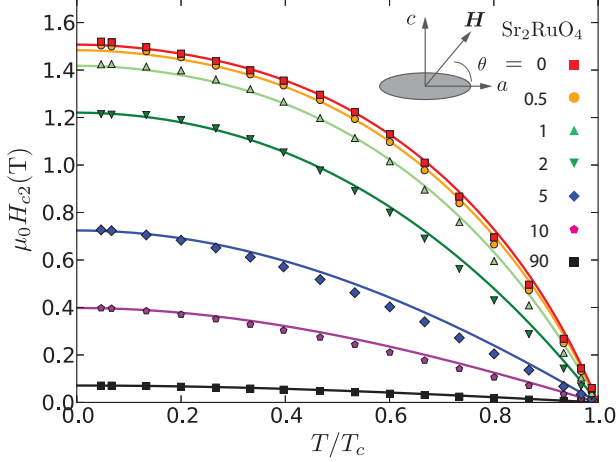


FIG. 3. Fits to the angular dependent  $H_{c2,a}(\theta, T)$  measurements of  $\text{Sr}_2\text{RuO}_4$  sample [13] using helical state (b) with  $\mathbf{d} = \hat{k}_x \hat{\mathbf{x}} - \hat{k}_y \hat{\mathbf{y}}$  with  $\bar{g}_{ab} = 1.9$ ,  $\bar{g}_c = 0.2$ .

We remark that all the helical states listed in figure 1 could equally well fit the data shown in figure 3, as the differences in their  $H_{c2}$  values only appear in their in-plane ( $\phi$ ) anisotropies. As seen from (1), in the absence of Pauli limiting,  $H_{c2,ab}(\phi)$  for the helical states are isotropic in the basal plane. However, with the fitting parameter  $\bar{g}_{ab} = 1.9$ , the  $H_{c2,ab}(\phi)$  at 0.13 K for the helical states (b) and (c) in figure 1 exhibit four-fold in-plane azimuthal anisotropies with a relative amplitude as large as 30% (figure 4(a)) and a phase shift of  $\pi/4$  between them, while those for states (a) and (d) remain isotropic in the  $ab$  plane. The observed in-plane anisotropy of  $H_{c2,ab}(\phi)$  is at most 3% and disappears either above 0.8 K or with a field misalignment of less than  $1^\circ$  [13, 19]. The calculated anisotropy for helical state (b) with  $\mathbf{d} = \hat{k}_x \hat{\mathbf{x}} - \hat{k}_y \hat{\mathbf{y}}$  state persists for  $T > T_c/2$  and for field misalignments greater than  $2^\circ$  (figure 4(b)). Thus, this parallel-spin  $p$ -wave state can explain the strong Pauli limiting for  $\mathbf{B} \perp \hat{\mathbf{c}}$ , but the details are not in precise agreement with the experimental observations [13, 19].

## DISCUSSION

A multi-component order parameter proposed to interpret the in-plane  $H_{c2,ab}(\phi)$  anisotropy in reference [19] turns out to have a similar problem of a large magnitude of the in-plane anisotropy [21]. There could also be two slightly misaligned crystals in the same sample [13], and the smaller region of the hysteretic magnetization data below 0.8 K in the more recent data of Yonezawa *et al.* than in the older Mao *et al.* and

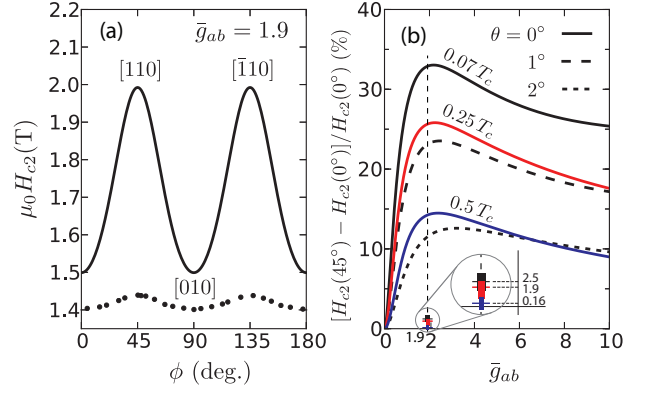


FIG. 4. In-plane  $H_{c2,ab}(\phi)$  anisotropy of helical state (b) with  $\mathbf{d} = k_x \hat{\mathbf{x}} - k_y \hat{\mathbf{y}}$ . (a)  $H_{c2,ab}(\phi)$  with an effective  $g$ -factor  $\bar{g}_{ab} = 1.9$  at 0.13 K. The amplitude of the predicted anisotropy (solid) is an order of magnitude larger than that (dotted curve) observed in  $\text{Sr}_2\text{RuO}_4$  by Mao *et al.* [19]. (b) Effects of  $g_{\text{eff}}$  to the relative magnitudes of the in-plane anisotropy at various temperatures and field misalignments. Anisotropies comparable to the experiments only occur with small  $\bar{g}_{ab}$  values. The symbols at the bottom represent the data of Kittaka *et al.* [13].

Deguchi *et al.* data are consistent with this scenario [12–14, 19]. Others think that this first-order transition below 0.8 K is more intrinsically due to a Fulde-Farrell-Larkin-Ovchinnikov state, entered below  $0.55T_c$  (close to 0.8 K in  $\text{Sr}_2\text{RuO}_4$ ) [45]. Based on the present calculations, if the Pauli pair-breaking effect is demanded as the source for the suppression on  $H_{c2,ab}$ , helical state (b) with  $\mathbf{d} = \hat{k}_x \hat{\mathbf{x}} - \hat{k}_y \hat{\mathbf{y}}$  has the same four-fold anisotropy with the same phase as in the experiments. Helical state (c) with  $\mathbf{d} = \hat{k}_y \hat{\mathbf{x}} + \hat{k}_x \hat{\mathbf{y}}$  has the four-fold anisotropy differing in phase by  $\pi/4$ . However, both of these azimuthal anisotropies are much stronger than that observed in experiment. However, the other helical (a) and (d)  $p$ -wave states with  $\mathbf{d} = \hat{k}_x \hat{\mathbf{x}} + \hat{k}_y \hat{\mathbf{y}}$  and  $\hat{k}_y \hat{\mathbf{x}} - \hat{k}_x \hat{\mathbf{y}}$  are predicted to have no azimuthal anisotropies at all. Including  $ab$ -planar anisotropy on the  $\gamma$  Fermi surface could lead to a small azimuthal anisotropy of  $H_{c2}(90^\circ, \phi, T)$ , but normally Fermi surface anisotropy is largest near to  $T_c$ . Thus, a single purported triplet-spin order parameter for  $\text{Sr}_2\text{RuO}_4$  is still elusive. We note, however, that there are many examples in which the Knight shift observations have been misleading and/or are also in apparent conflict with the upper critical field results [28, 46], strongly suggesting that a new theory of the Knight shift might lead to a possible resolution of the symmetry of the order parameter in  $\text{Sr}_2\text{RuO}_4$  [27, 28].

In summary, we studied the four helical  $p$ -wave states potentially realized in  $\text{Sr}_2\text{RuO}_4$  at  $H_{c2}$  by fitting the angular dependent  $H_{c2,a}(\theta, \phi, T)$  measurements, taking the Pauli paramagnetic effects into account by imposing strong spin-orbit coupling effects as the origin of the  $H_{c2,ab}$  suppression. In the ranges of the fitting param-

ters, one of the four helical states was predicted to have in-plane  $H_{c2}(90^\circ, \phi, T)$  four-fold azimuthal anisotropy with the same phase as observed, but both that azimuthal anisotropy and that from the (c) helical state with the anisotropy shifted by  $\pi/4$  in phase, had amplitudes that were predicted to be much stronger than that observed in  $\text{Sr}_2\text{RuO}_4$ . The  $H_{c2}(90^\circ, \phi, T)$  behaviors of the two other helical  $p$ -wave states were predicted to be completely independent of  $\phi$ , as long as in-plane Fermi surface anisotropy could be safely ignored. Other attempts to fit an order parameter such as  $\Delta_0[\sin(k_x a) + i \sin(k_y a)]$  with the low- $T$  specific heat  $C_V \sim T^2$  dependence failed to confront the very strong Pauli limiting of  $H_{c2}(90^\circ, \phi, T)$ [26]. Thus, the thermodynamic zero-field specific heat measurements appear to be in direct conflict with the field-dependent thermodynamic specific heat and magnetization measurements of the upper critical field[12–14, 19]. Further calculations to try to fit the excellent scanning tunneling microscopy results of Suderow *et al.* with a  $p$ -wave order parameter are also needed[25]. A point node in a helical  $p$ -wave order parameter might smear the sharp density of states walls they observed, but an accurate calculation is needed to quantify this possible disagreement.

## ACKNOWLEDGMENTS

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